Active Liquidity Management in Uniswap v3

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Abstract

UniswapV3 has been launched for more than 1 year and currently is the largest Decentralized Exchange(DEX) by trading volume. Compared to Uniswap V2,the key differentiating feature is the active liquidity management (ALM) where LPs can specify concentrated price ranges where they wanna provide liquidity in. It helps increase the capital efficiency but at same time making it more difficult for retail users to understand and make money due to the higher impermanent loss risk that comes with it.

In this work, we model the return and impermanent loss of a UniswapV3 LP to find the optimal position range that can maximize LP's return with given assumptions. Our model suggests active liquidity management strategies with liquidity being passive in volatile or trending markets can yield higher returns. We further build on our analysis to create an ALM strategy and back test on the historical data. We also take into account JIT liquidity which hampers LP's return but found that it contributes to less than 2% of the overall pool volume. A comparative study of major ALM protocols like Arrakis, Gamma during some period shows that our strategy would have surpassed their performance over the 2022's data.

1 INTRODUCTION

Decentralized Exchange(DEX), which supported by smart contracts, allows users to swap tokens without giving up self-custody. Crypto investors are paying more and more attention to DEX, especially after the fall of the gaint centralized exchange FTX in year 2022. Uniswap, currently the largest DEX on ethereum, contributes around 70% of the overall volume on DEX. It functions as Automatic Market Marker(AMM) with a Constant Product Market Maker(CPMM) algorithm. Originated from UniswapV2, UniswapV3 also applies CPMM method but with concentrated liquidity deployed. More specifically, UniswapV3 liquidity providers(LPs) need to specify the price range in which they wish to supply liquidity for. Their liquidity will only support upcoming swaps within the price range. Once the swap price go out of the price range, they will end up holding one singal tokens and stop collecting transaction fees utill price go back into the range again. By doing so, UniswapV3 provides an increased liquidity concentration around the current price. This will increase the capital efficiency for the makers while reduce the transaction slippage for the taker.
On the other hand, concentrated liquidity also increase the difficulty for LPs to optimize their market making strategy. In UniswapV2, LPs only need to decide which pool to provide liquidity and how much liquidity to provide. While in UniswapV3 both the price range and when to remove the position are the factor that LPs need to consider and will determine their returns correspondingly.

It is difficult for a LP to optimize their UniswapV3 positions given the higher Gamma risk that concentrated liquidity brings. Many studies have been focus on UniswapV3 and its return. Lioba et al.[1] present a study to illustrate the choices faced by UniswapV3 LPs and their implications. They explain the fundamentals of UniswapV3 in details and demonstrate how hard it is for a LP to stay profit, but do not provide a solution, especially for Non-stable tokens pool. In a series of blog posts ,Lambert[2][3]builds a theoretical model to guide LPs to find the optimal position. However, he only model the LPs' fee return and the impermanent loss is not factored in the model.

In this paper, we will work on both a theoretical model and a historical back-testing result to demonstrate our solution to optimize UniswapV3 positions.

2 THEORETICAL MODEL

In this section, we provide LP return formula and break it down into different components. We solve the formula step by step to link the return to the position range r we choose.

2.1 Assumptions and Parameters

To mint a new UniswapV3 position, after choose the pool and the total invest amount, the left choices for a LP are just the upper tick and lower tick. To simplify the expression of position range r, we assume the upper and lower boundary are centered around the current price. More specifically the starting price is the geometric mean of upper and lower boundary. Assume pools current price is P_0 and position range is $[P_l, P_u]$, we get following relations:

$$P_0 = \sqrt{P_l \cdot P_u} \qquad r = \sqrt{\frac{P_u}{P_l}} \qquad where \quad r > 1$$

$$P_u = P_0 \cdot r \tag{1}$$

$$P_l = \frac{P_0}{r} \tag{2}$$

After the minting the position, the following decision is when to close it. This decision is related to the current price and the position boundaries. Once the price move out of the position boundary, it will not collecting more transaction fees while may suffer more impermanent loss if price further move away. On the other hand, we can also decide to close a position early in order to avoid the accelerating Gamma loss around the boundary. In this work we assume the position only be re-balanced when the price first goes out of the position boundary in order to simply the calculation of positions fees and Impermanent Loss in the later part. Assume we close position at time T, the closing price P_1 will always equal to one of the boundary prices:

$$P_1 = P_l \qquad or \qquad P_1 = P_u \tag{3}$$

When modeling instrument price movement, people usually consider it a stochastic process and evolves randomly over time. In our work, we consider the Token price follows a Geometric Brownian Motion with constant Volatility σ and drift μ, the price is described by a stochastic partial differential equation given by:

$$dP_t = \mu P_t dt + \sigma P_t dW_t$$

Some assumptions about the Uniswap pool to simply the formula:

- Trading flow in the pool remain constant
- Liquidity distribution in the pool remain constant

2.2 LP fees

Transactions fees are the main revenue source for a LP. The fees are collected for each trade and then distributed to the LPs who has deposited liquidity within the traded price range. The total fees of a position should depend on the fee tier f, the trading volume per unit of time V_t, the total liquidity at the traded tick L_{total}, LP positions liquidity L and total amount of time the position within the range. In our case, because we assume the position will be closed once price move out of the range, we will have the estimated position holding time t, within which the price will always in the position boundary. Therefore in our formula:

$$LPfees = f\frac{V_t}{L_{total}}Lt$$

2.2.1 Fee accrual rate

We consider the first part of LP fees $f\frac{V}{L_{total}}$ as fee accrual rate, which is a pool's characteristic that describe the fee accumulating speed per active liquidity. In our assumption, both the trading flow and liquidity distribution are constant. Therefore for short to middle holding time, Fee accrual rate can be consider as a constant V_L in the formula

2.2.2 Position liquidity

UniswapV3 use the concept of virtual reserves, the contract track the liquidity L and sqrtPrice $\sqrt{P}$. if current price P_0 in price range $[P_l, P_u]$,we can find the following formula in white paper to get the virtual liquidity with real reserve x and y:

$$(x + \frac{L}{\sqrt{P_u}})(y + L\sqrt{P_l}) = L^2$$

Further solve the equation we can get:

$$x = L\frac{\sqrt{P_u} - \sqrt{P_0}}{\sqrt{P_0} \cdot \sqrt{P_u}} \tag{4}$$

$$y = L(\sqrt{P_0} - \sqrt{P_l}) \tag{5}$$

Assume the total investment value in token y is V, we have:

$$V = x \cdot P_0 + y$$

Substitute in equation (4) and (5):

$$\begin{aligned} V &= x \cdot P_0 + y \\ &= L(\frac{\sqrt{P_u} - \sqrt{P_0}}{\sqrt{P_u \cdot P_0}} P_0 + \sqrt{P_0} - \sqrt{P_l}) \\ &= L(\frac{\sqrt{P_0 \cdot r} - \sqrt{P_0}}{\sqrt{P_0 \cdot r \cdot P_0}} P_0 + \sqrt{P_0} - \sqrt{\frac{P_0}{r}}) \\ &= L\sqrt{P_0}(\frac{\sqrt{r} - 1}{\sqrt{r}} + 1 - \frac{1}{\sqrt{r}}) \\ &= 2L\sqrt{P_0}(\frac{\sqrt{r} - 1}{\sqrt{r}}) \end{aligned} \tag{6}$$

Transform equation (6) we have the liquidity formula with expression of r,V and P_0:

$$L = \frac{V\sqrt{r}}{2\sqrt{P_0}(\sqrt{r} - 1)} \tag{7}$$

2.2.3 Estimated holding time

In our case, the estimated holding time of the position is the expected time that price first hits either the upper or the lower boundary. Since we assume the price follow a Geometric Brownian Motion, the expected first hit time has solved by Wilmott [4]:

$$\begin{aligned} t &= \frac{1}{\frac{1}{2}\sigma^2 - \mu}(ln(P_0/P_l) - \frac{1 - (P_0/P_l)^{(1-2\mu/\sigma^2)}}{1 - (P_u/P_0)^{(1-2\mu/\sigma^2)}} ln(P_u/P_l)) \\ &= \frac{1}{\frac{1}{2}\sigma^2 - \mu}(ln(r) - \frac{1 - r^{1-2\mu/\sigma^2}}{1 - r^{2(1-2\mu/\sigma^2)}} ln(r^2)) \end{aligned} \tag{8}$$

The estimated holding time t is only affected by r, μ and σ.

2.3 Impermanent Loss

Impermanent loss is the main risk and the major cause of loss faced by LP. It describes the loss in value of a liquidity position in comparison to holding the original assets til now. From the white paper we have $x = \frac{L}{\sqrt{P}}, y = L \cdot \sqrt{P}$. Thus the Impermanent loss IL can be write as follow:

$$
\begin{aligned}
IL &= P_1 x_1 + y_1 - (P_1 x_0 + y_0) \\
&= P_1 \cdot \frac{L}{\sqrt{P_1}} + L\sqrt{P_1} - (P_1 \cdot \frac{L}{\sqrt{P_0}} + L\sqrt{P_0}) \\
&= 2L\sqrt{P_1} - \frac{L}{\sqrt{P_0}} \cdot P_1 - L\sqrt{P_0}
\end{aligned} \tag{9}
$$

Substitute in equation (3) and equation (7), we have:

$$
\begin{aligned}
IL &= L(2\sqrt{P_1} - \frac{P_1}{\sqrt{P_0}} - \sqrt{P_0}) \\
&= L(2\sqrt{P_0 \cdot r} - \frac{P_0 \cdot r}{\sqrt{P_0}} - \sqrt{P_0}) \\
&= L\sqrt{P_0}(2\sqrt{r} - r - 1) \\
&= L\sqrt{P_0}(\sqrt{r} - 1)(1 - \sqrt{r}) \\
&= \frac{V \cdot \sqrt{r}}{2\sqrt{P_0} \cdot (\sqrt{r} - 1)} \sqrt{P_0}(\sqrt{r} - 1)(1 - \sqrt{r}) \\
&= \frac{V}{2}\sqrt{r}(1 - \sqrt{r}) \\
&= -\frac{V}{2}\sqrt{r}(\sqrt{r} - 1)
\end{aligned} \tag{10}
$$

2.4 Re-balance Cost

Re-balance cost is the cost to move token holding to the initial amount after position withdrawal. Re-balancing is essential to make LP delta neutral after the position being removed. In this work, we assume the re-balance cost being fixed cost ratio c times the total Investment value V:

$$RebalanceCost = -c \cdot V \tag{11}$$

2.5 Turnover Times

Turnover times measures how many positions a LP can mint within a given investment time period. In this work, we let the total investment time be T, with the given estimated holding time per position calculated in equation (8), we have:

$$TurnoverTimes = \frac{T}{t} \tag{12}$$

2.6 Formula of LP return

Finally, to calculate the overall return of LP, denoted as R, we should consider all the component mentioned above:

$$R = \frac{(LPfees + IL + RebalanceCost)TurnoverTimes}{V}$$

Substitute in the equations detail above, we have:

$$\begin{aligned} R &= \frac{(f\frac{V_t}{L_{total}}Lt + IL - cV)\frac{T}{t}}{V} \\ &= (f\frac{V_t}{L_{total}}\frac{V\sqrt{r}}{2\sqrt{P_0}(\sqrt{r}-1)}t - \frac{V}{2}\sqrt{r}(\sqrt{r}-1) - cV)\frac{T}{t}\frac{1}{V} \\ &= \frac{fV_L\sqrt{r}T}{2\sqrt{P_0}(\sqrt{r}-1)} - \frac{\sqrt{r}(\sqrt{r}-1)T}{2t} - \frac{CT}{t} \end{aligned} \tag{13}$$

where

$$t = \frac{1}{\frac{1}{2}\sigma^2 - \mu}(ln(r) - \frac{1 - r^{1-2\mu/\sigma^2}}{1 - r^{2(1-2\mu/\sigma^2)}}ln(r^2))$$

In this equation, besides the range r, all other parameter can be estimated as some constant from historical data. Therefore, solve the equation to find the range r that maximize the return R will be our target.

3 METHODOLOGY

In previous section, we derived the Formula13, which shows how LP's return will be affected by the decision of position range r. The three part of the formula can be translated to Total fee, Total realized impermanent loss and Total re-balance cost. Although we cannot have a close form solution to find the range r that maximize the return R because of the difficulty to get the derivative, the curve of it do have a local maximum which is also the global maximum with constrain $r > 1$. Therefore simple numerical method can help us find the optimum value.
Besides, with Formula13 we can also find some useful characteristic to help us make decision in how to execute our LP strategy

Firstly, the optimum range r can neither be too small or too large. If r too small, the Total re-balance cost part increased significantly to eat all the revenue. On the other hand, if r too large, the Total fee part reduce significantly. Figure 1 shows how the curve will look like.

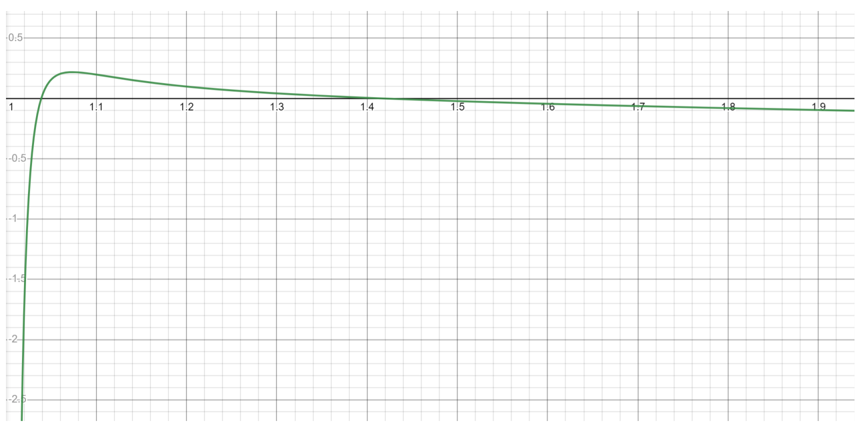
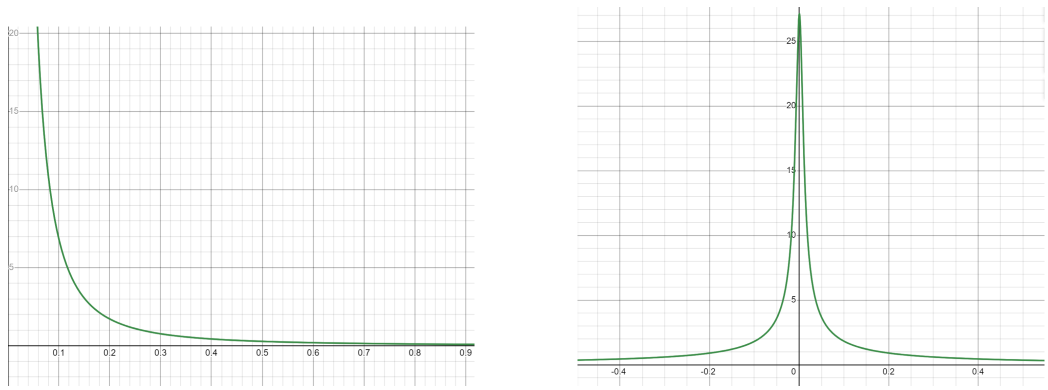


Figure 1: Sample plot of R relate to r

Secondly, in Formula13, when volatility σ or absolute value of drift $|\mu|$ increase, the estimated holding time t will decrease quickly. This causes both the Total realized impermanent loss and the Total re-balance cost parts increase, which reduce the LP return R. This suggest we should avoid providing liquidity in the high volatile or strong trending market. Figure 2 shows the indicative relation of estimated holding time t related to volatility σ and drift $|\mu|$ correspondingly.



(a) t relate to σ (b) t relate to μ

Figure 2: Sample plot of t

Lastly, the Total fee part is also sensitive to the Fee accrual rate V_L. When V_L decrease, the Total fee reduce significantly. This suggest we should pick the pool with higher Fee accrual rate at to provide liquidity. Figure 3 shows the indicative relation between LP return R and Fee accrual rate V_L.

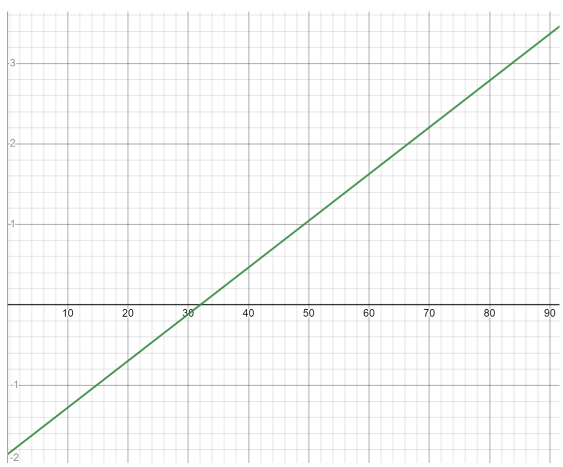


Figure 3: Sample plot of R relate to V_L

4 JUST-IN-TIME LIQUIDITY

JIT liquidity is a special form of liquidity provision where an LP mints and burns a concentrated position immediately before and after a swap. JIT LP will eat most of the transaction fees once its liquidity are deployed which will affect normal LP's fee return. To figure out the impact of JIT liquidity, we research on 3 different pool with different fee tiers, and the result as show in the table:

Pool	USDC/ ETH	USDC/ ETH	1INCH/ ETH
Fee tier	5bps	30 bps	100bps
No. Mint/Burn txns (7 Days)	2000	1400	10
JIT Liquidity occurrence	582	163	2
Token0 Volume (USDC)	73,113,220.60	17,005,240.77	35,322.20
Token1 Volume (ETH/1INCH)	57,660.30	13,892.94	66,457.55
7D Volume (USD)	8.33B	1.06 B	15.12 M
% of total volume	0.877%	1.60%	0.23%

Table 1: JIT liquidity volume

From Table 1 we can see that total trading volume sandwiched by JIT liquidity is not significant compare to overall volume for the past 7days data, only contributes below 2% among all the 3 pools. Some of the reasons may explain the situation are:

1. JIT liquidity requires strong MEV infrastructure, not available for most of the LPs
2. JIT liquidity needs to target on large enough trades to remain profitable
3. JIT liquidity ususally has lower profit margin for large trades compare to the arbitrage strategy. Arbitragers are willing to pay more gas fee, which JIT hardly can compete.

Given the lower weight of JIT volume among all the pool volume, we will not consider it a major problem for normal LPs performance.

5 COMPETITORS

In this section, we go through some of the major existing ALM protocol and anaysis there ALM algorithms.

5.1 Arrakis Finance

Arrakis v1 provided support for creating fungible vault wrappers over Uniswap V3 liquidity positions. One of the few reasons for success of Arrakis Finance vaults are:

1. It converts Uniswap v3 NFT LP positions into fungible G-UNI tokens.These G-UNI tokens are ERC20 which enable them to be staked similar to Uniswap v2 LP tokens for farming incentives.
2. They have built active connections with other projects to enable liquidity mining incentives for staking liquidity through Arrakis Finance.
3. They have got the G-UNI tokens for DAI-USDC Uniswap v3 pools to be enabled as collateral for MakerDAO vaults with liquidation temporarily disabled which adds leverage to the system and scales up TVL significantly.

In the current form, above 90% of Arrakis TVL is in soft pegged pairs which do not require active liquidity management. For a greater share of the remaining TVL, the protocol has kept fixed conservative ranges. The narrative has been more about a fungible ERC20 wrapper over Uniswap v3 LP positions and less about active liquidity management.

5.2 Gamma Finance

Gamma is a protocol designed for the non-custodial, automated, active management of concentrated liquidity pools. Gamma's core technology is a position manager contract called a Hypervisor. A Hypervisor has certain management functions such as rebalancing, position setting, and fee processing. These functions allow the Hypervisor to actively manage funds in a liquidity pool. Gamma is active on Ethereum, Optimism, Arbitrum, Polygon and Celo. Total TVL across all chain is $4.3M

The current price bands are [\$122, \$12,282] with ETH prices close to \$1,250. This follows similar patterns across all chains where the price bands are $\pm$10,000. Since the price has been trending within the bands, there haven't been many re-balances on their vaults.
Part of their design relies on a black-box close-sourced LP calculation engine which is supposed to calculate the optimal ranges for providing liquidity and feeding this data to the contracts but since there is no visibility on these calculations nor mention of their methodologies made public, it is hard for an LP to verify that Gamma is doing this in an optimal way.

6 RESULT

While reviewing other ALM's return, different protocols use different calculation method. To make the returns comparable, we use our own method to calculate the return for competitors. We consider the difference between final token balance's USD value and initial token balance's USD value at end time as the return. This will consider in both the transaction fees and impermanent loss.

With the formula and it's characteristics in previous sections, we pick USDC-ETH 0.0005 pool,which is the most active pool in uniswapV3 to do the back-testing of our own strategy. We control our strategy to stay passive when volatility is high or in a trending market. This is the plot of the strategy performance:

Figure 4: back-testing result from Apr 2022 - Dec 2022

As shown in Figure4, the upper plot is the back-testing return to the block number, the middle plot is the ETH price and corresponding UniswapV3 position range, and the lower plot is the realized volatility. In the back-testing period, our strategy has an annualized return of 12.31%, sharpe ratio of 2.65 and maximum draw down of 14.94%.

We also calculate the performance for Arrakis and Gamma in the same period. The return plot is shown in the Figure5 and the statistics in Table2.

Figure 5: Competitors Performance

Protocal	range	arrakis	gamma
return	12.31%	-55.33%	-52.78%
sharpe	2.65	-3.02	-3.07
Maximum Drawdown	-14.94%	-69.68%	-71.95%

Table 2: Competitors Performance

From the plot, we can see that both arrakis and gamma's return shows a high beta relation with ETH price. They are not controlling their delta risk exposure efficiently. Thus we outperform then both in the down trending market and the low-volatility market.

Current model is the most straightforward one, there are many potential improvements we can do in the future:

- Dynamic re-balance throughout the position period
- Automatic pool selection to filter out the profitable pools
- Hedging with options to perfectly hedge the Gamma risk

7 CONCLUSION

Providing liquidity to UniswapV3 and making profit from it is not a simple work, it requires comprehensive understanding of the UniswapV3 model and sophisticated risk management skills. LPs cannot avoid the risk of impermanent loss unless using JIT method. However the nature of JIT limit its access to retail users and the success chance. ALM becomes a good choice for retail users to join UniswapV3 as LP without worrying too much about its complexity. Given the ALM competitors are not optimizing the return, our strategy can help generate more profit not only in theoratical model but also in the backtesting result.

References

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